International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 12, December 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: **http://www.ijesm.co.in, Email: ijesmj@gmail.com**
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

INTERVAL -VALUED S-NORM SOFT G-MODULES OVER N-R VECTOR SPACE.

R.Thirunavukkarasu *& Dr.S.Subramanian**

**** Research Scholar, Department of Mathematics, PRIST University, Tanjore, Tamilnadu**

**** Professor, Department of Mathematics, PRIST University, Tanjore, Tamilnadu**

Abstract: we apply the concept of interval –valued fuzzy soft sets to near-ring modular structures. The idea of interval-valued S-norm soft G-modules has been explained through S-norm in near-ring modules and investigate some of their properties.

Keywords: Soft set, S-norm, soft G-modules, N-R vector space, level set, S-norm soft G-modules, interval valued set.

Section-1.Introduction:.In the Soft Set Theory, we have the opposite approach to this problem. Theinitialdescriptionoftheobjecthasanapproximatenature,andwedonotneed to introduce the notion of exact solution. This motivated D. Molodtsov's work [4] in 1999 titled Soft Set Theory first results. Therein, the basic notions of the theory of Soft Sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future with regard to the theory. Soft Set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. Maji et al. [1] put forward the concept of fuzzy soft sets, which is a hybrid model of fuzzy sets and soft sets. Membership function plays an important role in fuzzy set theory. Many new operations on soft sets were introduced by Maji et al [2, 3] in a later paper In 1999, Molodtsov [4] introduced the concepts of soft sets, which is parameterized family of subsets defined over a universe and a set of parameters as a model to capture uncertainty and vagueness in data. Many of Soft set theory have been discussed by Molodtsov in [4]. Tripathy et al [5] defined soft sets through their characteristic functions. This approach has been highly authentic and helpful in defining the basic operations of soft sets. Zadeh [9] initiated the concept of fuzzy sets in 1965 which is considered as generalization of classical or crisp sets. Similarly, it is expected that defining membership function for fuzzy soft sets will systematize many operations defined upon them as done in [7]. In this paper,we apply the concept of interval –valued fuzzy soft sets to near-ring modular structures. The idea of interval-valued S-norm soft G-modules has been explained through S-norm in near-ring modules and investigate some of their properties.

Section-2. Preliminaries

Definition 2.1: Let X be a non-empty set. A mapping A: X→ [0,1] is called a fuzzy set in X.

Definition 2.2: Let X be a set. A mapping M : X → D[0,1] is called an interval –valued fuzzy set (briefly i-v set) of X, where D[0,1] denotes the family of all closed subintervals of [0,1], and [M(x)] = [$M^-(x)$, $M^+(x)$], for all x ∊ X, where $M^-(x)$ and $M^+(x)$ are fuzzy sets in X.

The notion of an interval –valued S-norm was introduced by [9] as follows.

A mapping S from D [0,1] x D[0,1] → D[0,1] given by

$S(x^-, y^-) = (s(x^-, y^-), s(x^-, y^-))$ for all x^-, y^- ∊ D[0,1] is called interval S-norm.

Proposition 2.3: Let S be an interval s-norm on D[0,1].Then

(i) $S(x^-, y^-) = S(y^-, x^-)$

(ii) $S(x^-, x^-) = x^-$

(iii) if $x \geq y$ in D[0,1]. Then $S(x^-, z^-) \leq S(y^-, z^-)$ for all x^-, y^-, z^- ∊ D[0,1].

Definition 2.4: A soft set is a parameterised family of collection of all subsets of X.

Definition 2.5: An interval-valued fuzzy soft set [M] in a near-ring vector space Rover a field F is called an interval-valued S-norm soft G-modules if

(i) [M](ax + by) ≤ S ([M](x), [M](y))

(ii) [M](xn) ≤ [M](x) for all x,y,n ∊ R.

Example 2.6: Let R = {x,y,z,k} be a set with two binary operations as follows

1.1 +	**1.2 x**	**1.3 y**	**1.4 z**	**1.5 k**
1.6 x	**1.7 x**	**1.8 y**	**1.9 z**	**1.10 k**
1.11 y	**1.12 y**	**1.13 x**	**1.14 k**	**1.15 z**
1.16 z	**1.17 z**	**1.18 k**	**1.19 y**	**1.20 x**
1.21 k	**1.22 k**	**1.23 z**	**1.24 x**	**1.25 y**

1.26 •	**1.27 x**	**1.28 x**	**1.29 x**	**1.30 x**
1.31 x	**1.32 x**	**1.33 x**	**1.34 x**	**1.35 x**
1.36 x	**1.37 x**	**1.38 x**	**1.39 x**	**1.40 x**
1.41 x	**1.42 x**	**1.43 y**	**1.44 z**	**1.45 k**

Then (R, + , •) is a near-ring vector space. We define a fuzzy soft set [M] : R→ [0,1] by [M](z) = [M](k) = [0.4 , 0.6] >[M](y) = [0.2, 0.3] > [0.1, 0.2]. Then [M] is an interval –valued S-norm soft G-modules.

Remark 2.7: If a =1 and b =1 in the above definition- , we have [M](x-y) ≤ S([M](x), [M](y)) and [M](xn) ≤ [M](x) for all x,y , n ∊ R.

So we can prove if M is an interval –valued s-norm of a near-ring vector space R overa field F. Then $[M](0) \leq [M](x)$ for all $x \in R$.

For every $x \in R$, we have

$[M](0) = [M](x- x) \leq S([M](x), [M](x))$

$= [s(M^-(x), M^-(x)), s(M^+(x), M^+(x)]$

$= [M^-(x), M^+(x)] = [M](x)$.

Section-3. PROPERTIES OF INTERVAL-VALUED S-NORM SOFT G-MODULES

Proposition-3.1: If $\{ [M_i] / i \in \vee\}$ is a family of an interval –valued S-norm soft G-modules of a near-ring vector space R, then so is $\cap M_i$, where $i \in \vee$ is any index set.

Proof: Let $x,y \in R$. Then

$(\cap[M_i]) (ax + by) = \min \{[M_i](ax +by) / i \in \vee\}$

$\leq \min \{ S\{[M_i](x), [M_i](y\} / i \in v\}$

$= S \{\min\{[M_i](x) / i \in \vee \}, \min\{[M_i](y) / i \in \vee\}\}$

$= S\{ (\cap[M_i])(x), (\cap[M_i])(y)\}$, and for any $x,n \in R$, we have

$(\cap[M_i])(xn) = \min \{[M_i](xn)/ i \in v\}$

$\leq \min \{[M_i](x) / i \in v\}$

$= (\cap[M_i])(x)$. Hence $\cap[M_i]$is an interval-valued S-norm soft G-modules of R.

Proposition 3.2: Let R be a near-ring vector space. An interval -valued fuzzy set M in R is an interval –valued S-norm soft G-modules of R if and only if M^- and M^+ are soft G-modules of R.

Proof: Assume that M^- and M^+ are interval –valued S-norm soft G-modules of R and let $x,y \in R$.

Then $[M](ax +by) = [[M] (ax +by), M(ax +by)]$

$\leq [S(M^-(x), M^-(x)), S(M^+(x), M^+(y))]$

$= S(M^-(x), M^-(y))$ and for $x,n \in R$.

$[M](xn) = [M^- (xn), M^+(xn)] \leq [M^- (xn), M^+(xn)] = [M](x)$.

Hence [M] is an interval –valued S-norm soft G-modules of R.

Conversly, Suppose that [M] is an interval –valued S-norm soft G-modules of R. For any $x,y \in R$.

We have

$[M^- (ax +by), M^+(ax +by)] = [M] (ax + by)$

$\leq S \{M^-(x), M^-(y))$

$= S (M^-(x), M^+(x)), (M^-(y), M^+(y))$

International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 12, December 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$= [\ s(M^-(x), M^-(y)), s\ (M^+(x), M^+(y)]$

It follows that

$[M](ax + by\ ) \leq S(M^-(x), M^-(y))$ and

$[M](ax + by\ ) \leq S(M^+(x), M^+(y))$. For any x ,n∈ R , we have

$[\ M^-(x), M^+(x)\ ] = [M](xn) \leq [M](x) \leq [\ M^-(x), M^+(x)]$ and so $[M](xn) \leq M^-(x)$ and

$M^+(xn) \leq [M](x)$. Hence M^- and M^+ are interval –valued S-norm soft G-modules of R.

Theorem-3.3: An interval valued fuzzy set [M] in a near-ring vector space R is a interval –valued S-norm soft G-modules of R if and only if the non-empty lower level set L ([M] ; [α ,β]) is a soft G-module of r for any [α ,β] ∈ D[0,1].

Proof: Let us assume that [M] is a interval valued S-norm soft G-modules of R and [α ,β] ∈ D[0,1] be such that x,y∈ L ([M] ; [α ,β]). Then

$[M](ax + by) \leq S([M](x), [M](y))$

$\leq S([\alpha ,\beta] ,[\alpha ,\beta]\ ) = [\alpha ,\beta]$.

And so ax +by ∈ L ([M] ; [α ,β]). Let n ∈ R.Then

We have $[M](xn) \leq [M](x) = [\alpha ,\beta]$, for every x ∈ L ([M] ; [α ,β]) and so xn∈ L ([M] ; [α ,β]).

Thus L ([M] ; [α ,β]) is a soft G-modules of R.

Conversly, assume that L ([M] ; [α ,β]). Is a soft G-modules of R for every [α ,β] ∈ D[0,1].

Suppose that there exist x_0 , y_0∈ R such that

$[M](ax_0 + by_0\ ) \leq S([M](x_0)\ , [M](y_0))$.

Let $[M](x_0) = [\alpha ,\beta]$, $[M](y_0) = [\gamma\ ,\delta]$ and

$[M](ax_0 + by_0\ ) = [\Delta_1, \Delta_2]$. Then $[\Delta_1, \Delta_2] > S\ ([\alpha ,\beta]\ , [\gamma\ ,\delta]\ ) = (s\ ([\alpha ,\gamma]\ , s[\beta\ ,\delta]\ )$.

Using the given definition, we assume that $\Delta_1 > s\ (\alpha,\gamma)$ and $\Delta_2 > s\ (\beta\ ,\delta)$.

Let $[\ \lambda_1, \lambda_2] = 1/3\ ([M](ax_0 + by_0\ ) \leq S([M](x_0)\ , [M](y_0))$.

$= 1/3\ (\ [\ \Delta_1\ , \Delta_2) + (s\ ([\alpha ,\gamma]\ , s[\beta\ ,\delta]\ )$

$= [\Delta_1 + s(\alpha ,\gamma) / 3\ , \Delta_2 + s(\beta\ ,\delta) / 3]$

It follows that

$s(\alpha ,\gamma) < \ \Delta_1 + s(\alpha ,\gamma) / 3\ \ < \Delta_1$ and

$s\ (\beta\ ,\delta) < \lambda_2 = \Delta_2 + s(\beta\ ,\delta) / 3 < \Delta_2$

Therefore x_0,y_0∉L ([M] ; [α ,β]).

On the other hand, we notice that

$s(\alpha,\gamma) \geq \alpha$, $s(\alpha,\gamma) \geq \gamma$, $s(\beta,\delta) \geq \beta$, $s(\beta,\delta) \geq \gamma$,

we get

$[M](x_0) = [\alpha,\beta] \leq (s([\alpha,\gamma], s[\beta,\delta]) < [\lambda_1, \lambda_2]$ and

$[M](y_0) = [\gamma, \delta] \leq (s([\alpha,\gamma], s[\beta,\delta]) < [\lambda_1, \lambda_2]$. And so $x_0,y_0 \in L([M]; [\alpha,\beta])$.

It is contradiction that $L([M]; [\alpha,\beta])$ is a S-norm soft G-modules of R.

Also suppose that there exist x_0, $n_0 \in R$ such that

$[M](x_0n_0) = [\Delta_1, \Delta_2]$. Then

$[\Delta_1, \Delta_2] > [\alpha, \beta]$.

Let $[\lambda_1, \lambda_2] = 1/3\ ([M](x_0n_0) + [M](x_0))$. Then

$[\lambda_1, \lambda_2] = 1/3\ ([\Delta_1, \Delta_2] + [\alpha, \beta])$

$=[(\Delta_1 + \alpha)/3, (\Delta_2 + \beta)/3]$.

It follows that $\alpha < \lambda_1 = \Delta_1 + \alpha)/3 < \Delta_1$ and

$B < \lambda_2 = (\Delta_2 + \beta)/3 < \Delta_2$

So that $[\alpha, \beta] < [\lambda_1, \lambda_2] < [\Delta_1, \Delta_2] = [M](x_0n_0)$.

Therefore $x_0n_0 \notin L([M]; [\lambda_1, \lambda_2])$..

On the other hand, we get

$[M](x_0) = [\alpha, \beta] < [\lambda 1, \lambda 2]$ and so $x_0 \in L([M]; [\lambda_1, \lambda_2])$. It is a contradiction that $L([M]; [\lambda_1, \lambda_2])$ is a soft G-modules of R.

Hence [M] is an interval –valued S-norm soft G-modules of R.

Conclusion: Using lower level set, we give a characterisation of interval valued S-norm soft G-modules and investigate some of their properties. One can obtain the similar idea in the field of Neutrosophic soft G-modules.

References:

[1] P.K Maji, R. Biswas and A.R Roy, "Fuzzy Soft Sets", Journal of Fuzzy Mathematics, 9(3), (2001) pp.589-602

[2] P.K Maji, R. Biswas and A.R Roy, "An Application of Soft Sets in a Decision Making Problem", Computers and Mathematics with Applications, 44 (2002) pp.1007-1083

[3] P.K Maji, R. Biswas and A.R Roy, "Soft Set Theory", Computers and Mathematics with Applications, 45 (2003) pp.555-562

[4] D. Molodtsov, "Soft Set Theory - First Results", Computers and Mathematics with Applications, 37 (1999) pp.19-31

[5] B.K. Tripathy and K.R. Arun, "A New Approach to Soft Sets, Soft Multisets and Their Properties", International Journal of Reasoning-based Intelligent Systems, Vol. 7, no. 3/4, 2015, pp.244-253.

[6] B.K Tripathy, T.R Sooraj, and R K Mohanty, "A new approach to fuzzy soft set and its application in decision making", accepted in ICCIDM 2015.

[7]. X.B. Yang, T.Y. Lin, J.Y. Yang, Y. Li and D. Yu, "Combination of interval-valued fuzzy set and soft set", Computers & Mathematics with Applications 58 (3)(2009) 521-527

[8]. T.R Sooraj, RK Mohanty, B.K Tripathy, "A New Approach to Interval valued Fuzzy Soft sets and its Application in Group decision making", accepted in CDCS 2015, Kochi.

[9] L.A Zadeh, "Fuzzy sets", Information and Control, 8 (1965) pp.338-353